Mathematical Writing

Improving our mathematical writing is a primary objective of this course. At HMC, this means that the skills you learned in Writ 1 are critical in Math 131!

Proof writing can seem really daunting. Sometimes it can feel like there is a "right" or "wrong" way to proceed, and that you need to demonstrate rigor in a particular way. This is not the case, and my hope for you is that this course helps you break free from this limiting belief.

While there are general principles that make a proof "good," the wonderful thing about proof writing is that there is never only one right way to construct a proof. The proof is as unique as the person who writes it. Developing your own personal mathematical style is all part of the fun.

Prof. Heather's opinion on proof

The purpose of a proof is to illuminate a mathematical truth (the theorem, lemma, or corollary). It's an explanation of why something is true that gives us additional insight or appreciation. A good proof achieves these goals in a way that is clear, concise, and enjoyable to read.

How do we write a good proof? First and foremost, good mathematical writing follows the principles of good writing.

1. **Consider your audience**. The level of detail that you include, the notation and terminology that you use, and the formality or informality of your language should depend on who your intended reader is. A good rule of thumb for this course is that you write your proofs so that they would be clear and understandable to yourself two weeks prior (or a few weeks from now when you're studying for an exam).

2. **Be clear and concise**. In my experience, it is much more common for new proof-writers to make their proofs too long rather than too short. Try to make your arguments in a direct way and do not include extraneous information.

3. **Use active voice**. It is tempting to hide behind passive voice to make our writing sound more "objective," but the opposite is true: using active voice is an excellent way to make your writing direct and clear. Furthermore, active voice highlights that there is a person (or people) doing the mathematics. This is empowering for you as a mathematician and it helps us all remember that mathematics is a human enterprise. In mathematics, we often use "we" as the subject even if we are writing the proof by

ourselves. It might sound funny at first, but this is a way to acknowledge that the proof is a shared experience between the writer and the reader.

4. **Revise, revise, revise!** Writing a good proof is a process ... nobody writes a good proof the first time! As with any writing, proofs should have multiple drafts. You can and should edit and solicit feedback.

My colleague Francis Su has written some excellent guidelines for good mathematical writing that dive further into these points. Please read this handout before proceeding on any of your written assignments.

This study by Inglis and Aberdein suggests that appraisal of whether a proof is "beautiful," "intricate," "rigorous," or even "useful" is subjective. We will spend a lot of time in class constructing and critiquing proofs. It's possible that you, your classmates, and I will sometimes disagree on points of style. This is totally normal and it's all part of the process.