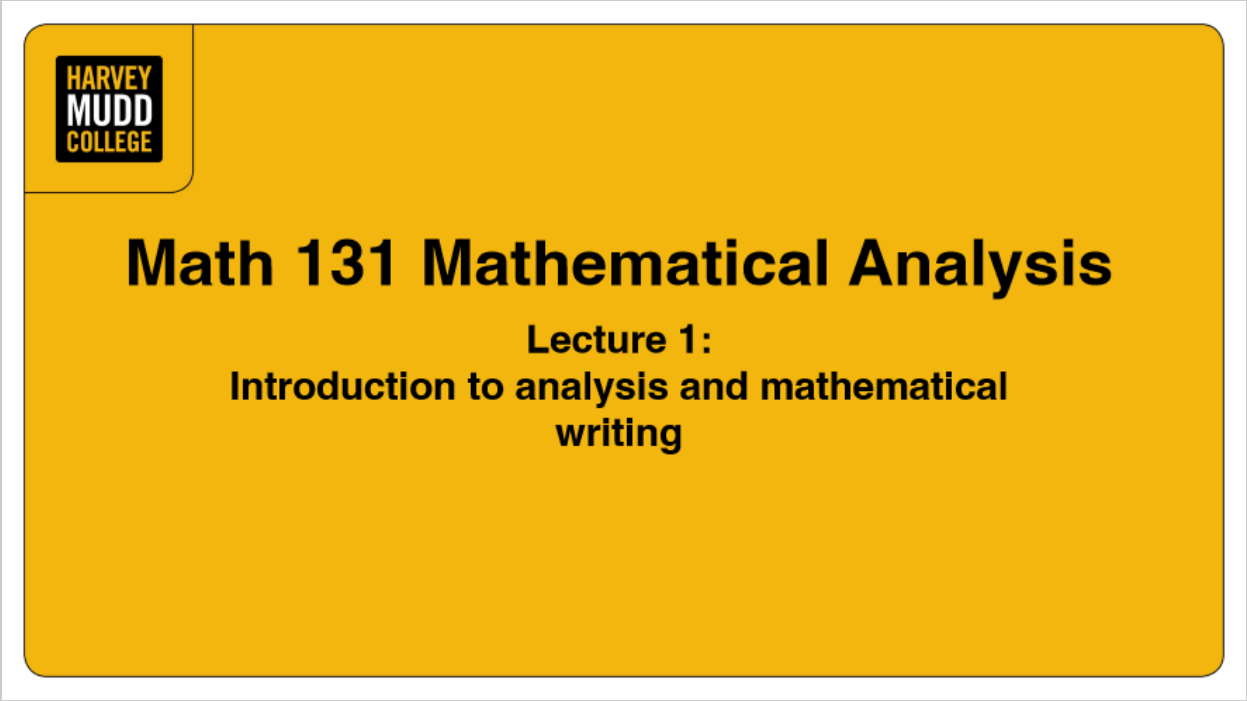
01-Lecture 1 Mathematical Writing_LdeP_S2026

Tuesday, January 20, 2026 12:56 PM

Lecture 1 Mathema...



Analysis is a class about infinity.

Part 1.

What does it mean for a set to be infinite?
What kinds of structure can infinite sets have?
How can we define distances and bounds?

Part 2.

How can we make sense of infinite lists and sums?

Part 3.

Parts 1 and 2 give us the tools to understand functions rigorously!

Intro: Course Tools and Canvas

https://harveymuddcollege.instructure.com/courses/3334

Learning Goal 1: Problem Solving

Problem-solving principles (Grant Sanderson)

- Use the defining features of a problem
- Start by solving the simplest possible variant
- Seek symmetry
- List any definitions/theorems/equations that might be useful
- Do examples to build intuition
- Draw pictures

Defining all objects
- Give names that make sense

Ex: Rectangle R
m, n: Natural #s
θ, α: Angles
ρ: Density

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Learning Goal 2: Formulate Mathematical Ideas well.

Table discussion

Let's solve a problem.

The problem: Think about rectangles with integer side lengths. Does every such rectangle with even area also have an even length side, and vice versa?

- Use the defining features of a problem
- Start by solving the simplest possible variant
- Seek symmetry
- List any definitions/theorems/equations that might be useful
- Do examples to build intuition
- Draw pictures

Common # sets:
$\mathbb{N}$: Natural #s
$\mathbb{Z}$: Integers
$\mathbb{Q}$: Rationals
$\mathbb{R}$: Reals
$\mathbb{C}$: Complex

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Learning Goal 2: Learn to formulate math ideas well.

Table discussion

Turning a problem into a conjecture

At the board: write your mathematically precise conjecture.

Think about rectangles with integer side lengths. Does every such rectangle with even area also have an even length side, and vice versa?

Write a conjecture based on your last discussion. Consider these tips:

- Define our mathematical objects of interest.
- What do we want to say?
- Use descriptive, full sentences.
- Don't editorialize.
- Revise as necessary.

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Turning our problem into a conjecture

Proof Structure

- Define objects
- Set up necessary assumptions
- State proposed implication

Think about rectangles with integer side lengths. Does every such rectangle with even area also have an even length side, and vice versa?

Conjecture. Let R be the set of all rectangles with integer length and width. Let $R_1 \subseteq R$ be the set of rectangles with even length or width, and let $R_2 \subseteq R$ be the set of all rectangles with even area. Then $R_1 = R_2$.

Now Prove your conjecture!

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Learning Goal 3: Improve math communication skills

Table discussion

Activity Part 1: What makes a good proof?

Read the four proofs of the rectangle theorem (on Canvas Module). **For each**, think about your reactions and give some feedback.

\+ • What do you like about each proof?
\- • What don't you like about each proof?
★ • Which is your favorite?

After reading these proofs:

- How would you revise **your** proof?
- What might you remove or incorporate?

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Table discussion

Activity Part 2: What makes a good proof?

Read the four proofs of the generalized triangle inequality (on Canvas Module). For **each**, think about your reactions and discuss the following questions.

- How would you rank order the proofs from "best" to "worst"?
- What made the worst proof worst?
- What made the best proof best?

Hint: • 2 proofs will lose points
• 2 proofs are correct, but one is the best.

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Proof 1: Incorrect induct. pf.
Not great formatting

Proof 2: Technically correct
Not pleasant to read.

Proof 3: The best! ★

Proof 4: Formatted well!
But: Ind. argument not correct!

Why write a proof?

The purpose of a proof is to illuminate a mathematical truth.

Some of the mathematical truths we take for granted today were not always accepted by the scientific community. Examples include:

- Existence of irrational numbers ~300 BC
- Existence of transcendental numbers Not alg. Late 1800's !

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Why write a proof?

The purpose of a proof is to illuminate a mathematical truth (the *theorem*, *lemma*, or *corollary*). It's an explanation of why something is true that gives us additional insight or appreciation. A good proof achieves these goals in a way that is **clear**, **concise**, and **enjoyable to read**.

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Before Next Class

Fill out the pre-class survey (link on Module 1 on Canvas)
Read and/or watch video on induction (Module 1 on Canvas)
Read and/or watch videos on countable and uncountable sets (Module 2 on Canvas)

Upcoming Deadlines

Homework 1: Sunday, 1 February 2026, before 10pm

Office Hours & Grutoring

TBD